14.

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I. N. S. B.

DISSERTATIO MATHEMATICA EXHIBENS FORMULAS GENERALES DE SOLIDIS REGULARIBUS,

QUAM,

Indulgente Amplissima Facult. Philosoph. in Regia ad Auram Academia,

PRÆSIDE

MARTINO JOHANNE WALLENIO,

MATHES. PROFESS. Reg. & Ord.
FAC. PHIL. p. t. DECANO.

Publice ventilandam sistit

ABRAHAMUS MECHELIN,
Wiburgensis.

Die XX. Decemb. Anni MDCCLXV.

L. H. Q. A. M. S.

ABOÆ,
Impressit JOH. CHRISTOPH. FRENCKELL.

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Concionatori Castrensi Legionis Savolaxiensis Vigilantissimo.

AVUNCULIS CARISSIMIS.

In stadio cursuque litterarum quamvis ad propositam metam difficilis sæpe sit contentio; attamen, obstantibus licet infortuniis quibusvis, remoram conatibus meis haud parvam injicientibus, exoptatissima jam, favente Numine, fruor occasione, qua multiplicata Vestra, Avunculi svavissimi, in me beneficia publice celebrare mihi liceat. Equidem quemadmodum, a teneris usque ungviculis, singulari amore me semper complexi estis, ita, crescente ætate, mearum rerum satagere haud gravati, adeo ut, qvibus in rebus opus erat, Vestra operâ consiliisque optimis mihi adesse non detrectaveritis. Cumque sic Vobis quam maxime devinctus explere haud valeam, quod Vestra mihi injunxerat benevolentia: mentem venerabundam gratissimamque &, quæ ejus erit character, exiguam hanc Dissertationem, Vobis cernuus offero. De cetero mihi nihil prius erit, nihil antiquius, quam S. Numen assiduis fatigare precibus, velit Vos, Avunculi svavissimi, omnigena felicitate beatos ad seram ætatem servare, ut sic in Vestrum Vestrorumque gaudium ac commodum vivatis, vigeatis, floreatis. Ero dum vixero

AVUNCULORUM CARISSIMORUM

cultor humillimus
ABRAH. MECHELIN.

Commissions Landtmätaren,

Ädel och Högaktad

Herr ERIC JOH. HAMMARIN,

MIN GUNSTIGE SVÅGER.

Det nära band, som oss, förmedelst et angenämt Svågerlag förknippar, jämte de många prof af ynnest och bevågenhet, jag i kraft däraf förspört, försätta mig i en oundvikelig förbindelse at härjämte tilskrifva Eder, min Gunstige Svåger, närvarande lilla arbete, innehållande några generela satser om Regulierа kroppar. Et ämne, som, så vida vitterligit är, ej blifvit tilförene afhandladt. Uptag altså detta, Min Gunstige Svåger, såsom en säker underpant af min skyldiga tacksamhet och anse benägit des välmening, som med önskan af all andelig och lekamlig sällhet förblifver

MIN GUNSTIGE SVÅGERS

ödmjuke tjenare
ABRAH. MECHELIN.

Perquam Reverendo atque Doctissimo

Domino JOHANNI MECHELIN,

Sacellano in Jockas meritissimo,

PARENTI INDULGENTISSIMO.

Exsplenduit tandem serena felicitatis aurora, sub qua, favente Numine, exercitio quodam Academico tenues meas ingenii vires periclitari sum ausus. Has proinde partes aggressurus nullus dubitavi, quin opellam hanc, Tibi, Parens Indulgentissime, jure proximo ac præcipuo, omni, qua par est reverentia, commendarem. Si enim quid unquam me Tibi reddit obstrictum, certe maxima & innumera, quæ ab ipsis incunabulis in me collata voluisti, beneficia. Quanta enim cura, quanto studio ac fervore in salutem meam & litterarum præcipué culturam incubuisti, id satis mea eloqui non valet lingua. Nulla certe aut injuria temporis aut etiam existente inopia rerum hunc ardorem defervere sivisti, sed e contrario votis, consiliis saluberrimis factisque ad id jugiter contendisti, ut prospera mea tandem evadant fata. Cumque tot tantisque beneficiis rependendis me imparem prorsus deprehendam simulque norim, Te, Parens Indulgentissime, tam sumtuum onere quam aliis ærumnis maxime gravatum, quibus ingentem cumulum adjecit festina ac inopinata Fratris mei carissimi non ita pridem defuncti mors: exiguum hoc chartaceum munus, in leve pignus gratæ mentis ac quoddam animi oblectamentum, Tibi venerabundus offero, quod ut benigne excipias etiam atque etiam rogo. Ceterum, quamdiu in vivis sum, ad DEum T. O. M. calidissimas fundam preces, velit Te, Parens Optime, sub variis hujus seculi molestiis, nobis clementer sustentare ac, post longam annorum seriem, sempiterno gaudio beare. Sic vovet ad cineres usque permansurus

PARENTIS INDULGENTISSIMI

filius obedientissimus
ABRAH. MECHELIN.

יהוה אדני הושיעה

§. I.

Theoria Corporum Regularium argutior quidem & elegantior quam utilior hodie habetur; prætereaque ab antiquioribus jam Geometris, præcipue EUCLIDE, HYPSICLE, PAPPO, ſcite adeo eſt exculta, ut quod addendum ſit, vix ſupereſſe videatur (*). Quia tamen ad perfectionem cujusque Scientiæ hoc omnino pertinere exiſtimandum, ut veritatibus ſeu propoſitionibus inſtructa ſit valde univerſalibus, plures particulares compendioſe complectentibus; maxime ſi illæ ita fuerit conceptæ, ut hae inde deduci commode queant:

(*) De iis tamen Seculo XVII:o copioſe commentatus eſt FRANCISCUS de FOIX de CANDALLE; atque de Cubo & Octaëdro, altero alteri inſcribendo, ſumta occaſione ex *EUCL. Elem. L. XV. Propp.* 3, 4, MAIRAN in Memoires de l'Acad. R. des Sc. de Paris 1725, quibus ſimilia in *Prop. 5:tam* ejusque converſam dici poſſent.

forte nec prorsus supervacaneum erit nec cultoribus Geometriæ ingratum, generales quasdam Formulas de Corporibus Regularibus breviter proponi, hactenus quidem, quod sciam, a nemine vel excogitatas vel publici juris factas (**).

§. II.

Esto $N =$ numero Hedrarum seu Figurarum planarum Regularium, quibus continetur solidum Regulare, quod dicatur 𝔄; $n =$ numero laterum cujusvis hedræ; $m =$ numero angulorum planorum, quemlibet solidum ipsius corporis comprehendentium; H dimidiam superficiem Sphæræ, circa corpus descriptæ; atque R angulum rectum denotent. Jam latera corporis seu dictarum hedrarum sunt chordæ circulorum maximorum Sphæræ, horumque arcus superficiem sphæræ dividunt in Figuras Sphericas regulares & æquales, quarum numerus $= N$; ut adeo harum quælibet sit $= \frac{2H}{N}$. Est quoque (*) eadem $= \frac{nV - (n-2)R}{2R} H$, designante 2V ali-

(**) Nisi forte aliqua ad præsentem materiam spectantia contineat Dissertatio Novis Comment. Petrop. Tom. IV, inserta EULERiana, quam videre nobis quidem haud contigit, sed cujus argumentum Novellæ Litterariæ Göttingenses (*Gött. Anzeigen von Gelehrten Sachen* 1763. p. 570.) his reddunt verbis: *Die Gründe einer erweiterten Lehre von den vieleckichten Körpern* (solidorum) *in ansehung der Anzahl der Flächen, der flachen Winckel und spitzigen Winckel* (ang. solidorum).

(*) Vid. in Actis Reg. Acad. Scient. Svec. Anni 1763. Trimestr. I. Dissertatio Præsidis: *Försök at mäta Hörn eller So-*

2V aliquem ipſius angulum ſphæricum. Unde ſequitur eſſe $V = R - \frac{N-2}{Nn} . 2R$. Et quia circa commune in ſuperficie Sphæræ punctum, quemlibet ſcilicet corporis apicem, jacent anguli ſphærici numero m, ſinguli $= 2V$: erit $2mV = 4R$, ſeu $V = \frac{2R}{m} = R - \frac{N-2}{Nn} . 2R$. Hinc oritur æquatio (I) $2Nn + 2Nm = Nnm + 4m$, relationem ſiſtens numerorum N, n, m, ſeu ex duobus eorum datis tertium invenire docens. Reperitur ſcilicet $N = \frac{4m}{2(m+n) - mn}$; $n = \frac{2m(N-2)}{(m-2)N}$; $m = \frac{2Nn}{Nn - 2(N-2)}$. Adhæc poſitis $M =$ numero angulorum ſolidorum, & $K =$ numero laterum corporis: facile intelligitur (**) eſſe (II) $Nn = Mm$, nec non (III) $Nn = 2K$; adeoque datis duobus quibuscunque horum quinque numerorum N, n, M, m, K, reliquos, ope trium iſtarum æquationum (I. II. III.) facile determinari. Etiamnum vero, quantum mihi quidem conſtat, deſideratur, nec inventu facilis videtur æquatio, quam non niſi duo dictorum quinque numerorum ingrediantur.

SCHOL. Si centra ſingularum hedrarum contiguarum corporis 𝔄 jungantur lineis rectis: haud difficulter patet deſcriptum iri in 𝔄 corpore aliud Corpus

lida Vinklar §. 6 vel 7; eſt ſcilicet ibi $W = 2nV$, vel $Y = R - V$.

(**) cfr. ſis EUCL. Elem. L. XV. Prop. 6.

Regulare $\mathfrak{B}$, habiturum *N* angulos solidos, singulos planis *n* comprehensos, hedras autem *M* & quidem *m* lateras. (cfr. æquat. II.). Si igitur possibile est corpus regulare speciei $\mathfrak{A}$, possibile etiam erit corpus regulare speciei $\mathfrak{B}$, atque his corporibus permutati competent valores numerorum N, M, simulque *n*, *m*. Sic comparata sunt *Hexaëdrum* & *Octaëdrum*, *Icosaëdrum* & *Dodecaëdrum*. Specie autem non differunt $\mathfrak{A}$ & $\mathfrak{B}$, si alterutrum fuerit *Tetraëdrum*; quia in hoc casu $M = N$, $m = n$.

§. III.

Sumto angulo acuto X, cujus Sinus $= \dfrac{\text{Cos.}\,\frac{2R}{m}}{\text{Sin}\,\frac{2R}{n}}$ existente sinu toto $= 1$: erit (*) *Inclinatio Hedrarum* $= 2X$, & *Angulus* corporis *solidus* $= \left(\dfrac{mX}{R} - m - 2\right) 2\mathfrak{R}$, designante $\mathfrak{R}$ angulum solidum rectum i. e. tribus angulis planis rectis contentum, qualis scilicet est angulus solidus Cubi seu Parallelipipedi recti. Continetur enim angulus corporis, *m* planis angulis, quorum quilibet, utpote pertinens ad figuram regularem rectilineam *n* laterum, est $= \dfrac{n-2}{n} . 2R$; dicta autem inclinatio æquipollet angulo Figuræ cujusdam Sphæricæ regularis, cujus singula latera, numero *m*, metiuntur hos ipsos totidem angulos planos. Constat ergo propositum (**).

Co-

(*) cfr. iis ibid. prop. 7.

(**) Per §§. 1. 7. Dissertationis supra (not. (*) §. II.) citatæ, COROLL. Quia, si latera corporum 𝔄 & 𝔅 (§. 2. SCHOL.) dimidia dicantur a & b respective, b erit cathetus oppositus angulo X in Δlo rectangulo, cujus hypothenusa $= a$. Cotang $\frac{2R}{n}$, radio scilicet circuli in hedra corporis 𝔄 inscribendi: sequitur Latus corporis 𝔄 esse ad latus corporis 𝔅 dicto modo inscripti, ut Sin $\frac{2R}{n}$. Tang $\frac{2R}{n}$ ad Cos $\frac{2R}{m}$ = (§. 2.) (Sin $\frac{N-2}{Nn}$. 2R).

§. IV.

Si arcus circuli maximi, puta latus cujusvis Figurarum Sphæricarum supra (§. 2.) memoratarum, dicatur $2L$: erit latus corporis regularis $= 2 \, Sin \, L$, posito Sphæræ radio=1. At(*) Cos L = $\frac{\text{Cos} \frac{2R}{n}}{\text{Sin V}}$ = (§. 2.) $\frac{\text{Cos} \frac{2R}{n}}{\text{Cos}\left(\frac{N-2}{Nn}.2R\right)}$; quare $\overline{\text{Sin L}}^2 = \frac{\overline{\text{Cos}\left(\frac{N-2}{Nn}.2R\right)}^2 - \left(\text{Cos}\frac{2R}{n}\right)^2}{\overline{\text{Cos}\left(\frac{N-2}{Nn}.2R\right)}^2}$. Cujus valoris numerator ut aptius exprimatur: ad tenorem subne-

in locum ipsorum n, L, V; Y, substitutis m, $\frac{n-2}{n}$. 2R, X, R—X; & quia ipsius 𝔑 mensura est ¼ H.

(*) Ibid. §. 7.

nexi Lemmatis(**) statuantur $\beta = \frac{N-2}{Nn}.2R$ & $\alpha = \frac{2R}{n}$, unde $\alpha+\beta = \frac{N-1}{Nn}.4R$, $\alpha-\beta = \frac{4R}{Nn}$, adeoque numerator $= \text{Sin}\,\frac{4R}{Nn}.\ \text{Sin}\left(\frac{N-1}{Nn}.4R\right)$. Ergo *Sin L* i. e. *Latus* vel dimidium vel integrum *Corporis Regularis*, prout vel radius vel diameter Sphæræ circumscriptæ fuerit $=1$, fit $= \frac{\sqrt{\text{Sin}\,\frac{4R}{Nn}.\ \text{Sin}\left(\frac{N-1}{Nn}.4R\right)}}{\text{Cos}\left(\frac{N-2}{Nn}.2R\right)} = \frac{\sqrt{\text{Cos}\left(\frac{m+n}{mn}.2R\right).\ \text{Cos}\left(\frac{m-n}{mn}.2R\right)}}{\text{Sin}\,\frac{2R}{m}}$. Prodit videlicet poste-

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(**) *Designantibus* α, β *duos arcus circulares seu angulos, erit* $\overline{\text{Sin}\,\alpha}^2 - \overline{\text{Sin}\,\beta}^2$ *vel* $\overline{\text{Cos}\,\beta}^2 - \overline{\text{Cos}\,\alpha}^2 = \text{Sin}\,\overline{\alpha+\beta}.\ \text{Sin}\,\overline{\alpha-\beta}$. Sunto (*Fig.* 1.) AB $=\alpha$ & AD $=\beta$ arcus circuli FAG, cujus centrum C. Duc rectas AC, DCG; atque BEKF perpendicularem ipsi AC; BH, FI ipsi DG; GL ipsi BF: erit BD $= \alpha+\beta$, cujus Sinus $=$ *BH*, DF $= \alpha-\beta$, ejus Sinus $=$ *FI*, Cos $\beta \pm$ Cos $\alpha =$ CM $\pm$ CE $=$ *GL* & *EM*. Jam ob æquiangula $\Delta\Delta$la BHK, CEK, FIK, GLK, oportet esse BH : BK :: CE : CK :: EM : KD seu BH : EM :: BK : KD, & FI : GL :: FK : KG; unde BH.FI : GL.EM :: BK.KF : DK.KG, atque ob BK.KF $=$ DK.KG, erit BH.FI $=$ GL.EM, i. e. $\text{Sin}\,\overline{\alpha+\beta}.\ \text{Sin}\,\overline{\alpha-\beta} = \overline{\text{Cos}\,\beta+\text{Cos}\,\alpha}.\ \overline{\text{Cos}\,\beta-\text{Cos}\,\alpha} = \overline{\text{Cos}\,\beta}^2 - \overline{\text{Cos}\,\alpha}^2 = \overline{\text{Sin}\,\alpha}^2 - \overline{\text{Sin}\,\beta}^2$. Q. E. D.

posterior hæc formula, sive in priori pro N substituatur ejus valor per m & n expressus (§. 2.) dein vero, ut ipsa paulo evadat concinnior, in locum ipsorum Sin $(\frac{m \pm n}{mn}.2R \mp R)$ & Cos$(R - \frac{2R}{m})$ subrogentur æquipollentes Cos$(\frac{m \pm n}{mn}.2R)$ & Sin $\frac{2R}{m}$; sive potius ne adhibita quidem formula priori, statuatur Cos L

$$= \left(\frac{\mathrm{Cos}\frac{2R}{n}}{\mathrm{Sin}\,V} = \frac{\mathrm{Cos}\frac{2R}{n}}{\mathrm{Sin}\frac{2R}{m}} = \right) \frac{\mathrm{Cos}\frac{2R}{n}}{\mathrm{Cos}(\frac{m-2}{m}.R)} \text{ vel } \frac{\mathrm{Sin}(\frac{n-2}{n}.R)}{\mathrm{Sin}\frac{2R}{m}},$$

atque cætera fiant similiter ac initio hujus §.

Coroll. Latera Corporum Regularium communi Sphæræ inscriptorum, in quibus m & n permutatos habent valores (§. 2. Schol.), erunt inverse ut Sin $\frac{2R}{m}$.

§. V. *Fig.* 2.

Vel ut paullo generalius simulque a primis quasi principiis rem repetamus: comprehendant circulorum maximorum arcus AbB = AcC = 2L angulum Sphæricum bAc = 2V; erunt chordæ AB = AC = 2 Sin L efficiuntque angulum rectilineum BAC, quem dico 2v. Esto M centrum Sphæræ; plana igitur ABb, ACc secabunt sese in recta MA. Ductis CD perpendiculari ad AM, DE ad BC, MG ad AB, & junctis DB, AE; anguli ADB AEB recti sunt, BDE = ½ BDC = V, BAE = v, △AGM ∽ △ADB; quare AM [= 1]: MG (= Cos L)::

[AB:

$\left[AB:BD::\frac{BE}{BD}:\frac{BE}{AB}::\right]$ Sin V: Sin v, adeo ut Cof L $=\frac{\text{Sin } v}{\text{Sin V}}$.

Transeundo jam ad corpora Regularia, est (§§. 2. 3.) $V=\frac{2R}{m}$ & $v=R-\frac{2R}{n}$; unde reperitur posterior formularum (§. 4.) exhibitarum, quæ ope æquationis I (§. 2.) in priorem, si ita visum fuerit, transformabitur.

§. VI. *Fig.* 2.

Recta perpendicularis [MO =] P a centro M Sphæræ vel Corporis Regularis ad aliquam ejus hedram ducta, in centrum O cadit planæ hujus figuræ regularis vel circuli circa eam descripti, cujus radius OA $=$ $\left[\frac{AG}{\text{Cof BAE}}=\right]\frac{\text{Sin L}}{\text{Cof } v}$. Jam in rectangulo △lo AOM est

$$MO^2=MA^2-AO^2=1-\left(\frac{\text{Sin L}}{\text{Cof } v}\right)^2=\frac{\overline{\text{Cof } v^2}-\overline{\text{Sin L}^2}}{\text{Cof } v^2}=$$

$$\frac{\overline{\text{Cof L}^2}-\overline{\text{Sin } v^2}}{\text{Cof } v^2}=(\S.\ 5.)\ \frac{\left(\frac{\text{Sin } v}{\text{Sin V}}\right)^2-\overline{\text{Sin } v^2}}{\text{Cof } v^2}=$$

$$\frac{\overline{\text{Sin } v^2}.(1-\overline{\text{Sin V}^2})}{(\text{Cof } v.\ \text{Sin V})^2}=\left(\frac{\text{Sin } v.\ \text{Cof V}}{\text{Cof } v.\ \text{Sin V}}\right)^2=\left(\frac{\text{Tang } v}{\text{Tang V}}\right)^2.$$ Ergo

$$P=\frac{\text{Tang } v}{\text{Tang V}}=(\S.\ 5.)\frac{\text{Cotang }\frac{2R}{n}}{\text{Tang }\frac{2R}{m}}=\frac{\text{Tang }\left(\frac{N-2}{Nn}.\ 2R\right)}{\text{Tang }\frac{2R}{n}}=$$

Semidiametro Sphæræ inscriptæ.

COR.

Cor. I. Diameter Sphæræ inſcriptæ eſt ad diametrum circumſcriptæ :: Cotang $\frac{2R}{n}$: Tang $\frac{2R}{m}$, vel Cot $\frac{2R}{m}$: Tang $\frac{2R}{n}$, vel 1 : Tang $\frac{2R}{m}$. Tang $\frac{2R}{n}$.

Cor. II. Hexaëdrum & Octaëdrum, æqualibus Sphæris inſcripta, equales habent *P*; pariterque (*) Dodecaëdrum & Icoſaëdrum. Permutatis enim *m*, *n*, haud mutatur valor ipſius *P*. cfr. §. 2. Schol.

Cor. III. Si hæc corpora eidem Sphæræ inſcribantur, circa communem Sphæram circumſcribi quoque poterunt, & viciſſim. (Cor. 2.)

Cor. IV. Atque Soliditates eorum erunt in ratione Superficierum.

§. VII.

Quia Figura regularis rectilinea, cujus latera ſingula, numero *n*, ſunt $= 2d$, habet aream $= \frac{ndd}{\text{Tang}\,\frac{2R}{n}}$: poſita ſemidiametro Sphæræ $= 1$, erit (§. 4.) *Superficies Corporis Regularis inſcripti* $= \frac{Nn.\ \text{Sin}\,\frac{4R}{Nn}.\ \text{Sin}\left(\frac{N-1}{Nn}.\,4R\right)}{\text{Tang}\,\frac{2R}{n}.\ \text{Coſ}\left(\frac{N-2}{Nn}.\,2R\right)^2}$;

 quæ

(*) Cfr. EUCL. Elem. L. XIV. Prop. 6. Demonſtr. coll. prop. 2. item PAPPI Collect. Mathem Lib. V. Prop. 48.

quæ ducta in $\frac{1}{3}$ *P* (§. 6.) dat *Soliditatem* $= \frac{Nn}{3}$.

$$\frac{\operatorname{Sin}\frac{4R}{Nn}.\operatorname{Sin}\left(\frac{N-1}{Nn}.4R\right).\operatorname{Tang}\left(\frac{N-2}{Nn}.2R\right).}{\left[\operatorname{Tang}\frac{2R}{n}.\operatorname{Cos}\left(\frac{N-2}{Nn}.2R\right)\right]^2}$$

§. VIII.

Cum sint Latera, Superficies, Soliditates corporum similium, communi Sphæræ inscripti atque circumscripti, in ratione simplici, duplicata, triplicata ipsius P : 1: facile reperiuntur (§§. 4. 7. 6.) *Corporis Regularis circa Sphæram, cujus semidiameter* = 1, *descripti Latus* =

$$\frac{2\operatorname{Tang}\frac{2R}{n}}{\operatorname{Sin}\left(\frac{N-2}{Nn}.2R\right)}\sqrt{\operatorname{Sin}\frac{4R}{Nn}.\operatorname{Sin}\left(\frac{N-1}{Nn}.4R\right)} = \frac{2\operatorname{Tang}\frac{2R}{n}.}{\operatorname{Cos}\frac{2R}{m}}$$

$$\sqrt{\operatorname{Cos}\left(\frac{m+n}{mn}.2R\right).\operatorname{Cos}\left(\frac{m-n}{mn}.2R\right)},$$ *Superficies* = Nn.

$$\frac{\operatorname{Tang}\frac{2R}{n}.\operatorname{Sin}\frac{4R}{Nn}.\operatorname{Sin}\left(\frac{N-1}{Nn}.4R\right)}{\operatorname{Sin}\left(\frac{N-2}{Nn}.2R\right)^2} = \textit{Solidati}\text{ triplæ.}$$

De-

§. IX.

Denique posito *Latere Corporis Regularis* $=1$, erit *Diameter Sphæræ circumscriptæ* $= \dfrac{\text{Sin}\,\frac{2R}{m}}{\sqrt{\text{Sin}\,\frac{4R}{Nn}.\,\text{Sin}\left(\frac{N-1}{Nn}.\,4R\right)}}$, *inscriptæ* $= \dfrac{\text{Cos}\,\frac{2R}{m}.\ \text{Cotang}\,\frac{2R}{n}}{\sqrt{\text{Sin}\,\frac{4R}{Nn}.\ \text{Sin}\left(\frac{N-1}{Nn}.\,4R\right)}}$; *Superficies Corporis* $= \frac{1}{4}\,Nn.\ \text{Cotang}\,\frac{2R}{n}$, *Soliditas* $= \frac{Nn}{24} \times \dfrac{\text{Cos}\,\frac{2R}{m}.\ \left(\text{Cotang}\,\frac{2R}{n}\right)^2}{\sqrt{\text{Sin}\,\frac{4R}{n}.\ \text{Sin}\left(\frac{N-1}{Nn}.\,4R\right)}}$

§. X.

Formulæ adductæ in alias, si placet, facile convertentur, duobus quibuslibet numerorum N, n, M, m, K, quamvis non omnes æque concinne, exprimendas (§. 2). Non autem nisi adscita theoria dimensionis Figurarum Sphæricarum (§. 2. not. (*)) vel æquatio I (§. 2.) vel pleræque Formulæ, quas duo tantum dictorum numerorum ingrediantur, inveniri posse videntur.

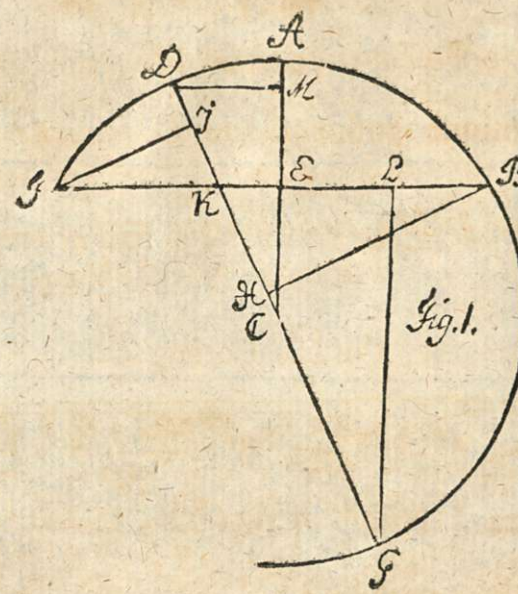

Fig. 1.

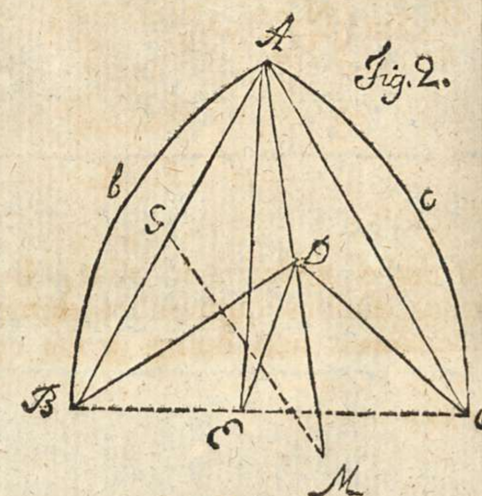

Fig. 2.

J. A. H. sculp.

Cæterum ex Formulis noſtris conſtructiones pro quovis Solido Regulari derivari poſſent. Præterea calculus horum Corporum in numeris determinatis, maxi metum expeditus tum adcuratus inſtituetur per logarithmos, ad ductum iſtarum Formularum facile ex Tabulis Sinuum & Tangentium artificialium eliciendos.